Material Model 24: Piecewise Linear Isotropic Plasticity

This plasticity treatment in this model is quite similar to Model 10, but unlike 10, it includes strain rate effects and does not use an equation of state. Deviatoric stresses are determined that satisfy the yield function

$$\phi = \frac{1}{2} s_{ij} s_{ij} - \frac{\sigma_y^2}{3} \leq 0 \quad (19.10.1)$$

where

$$\sigma_y = \beta \left[ \sigma_y + f_h \left( \varepsilon_{\text{eff}}^p \right) \right] \quad (19.24.1)$$

where the hardening function $f_h \left( \varepsilon_{\text{eff}}^p \right)$ can be specified in tabular form as an option. Otherwise, linear hardening of the form

$$f_h \left( \varepsilon_{\text{eff}}^p \right) = E_p \left( \varepsilon_{\text{eff}}^p \right) \quad (19.10.3)$$

is assumed where $E_p$ and $\varepsilon_{\text{eff}}^p$ are given in Equations (19.3.6) and (19.3.7), respectively. The parameter $\beta$ accounts for strain rate effects. For complete generality a table defining the yield stress versus plastic strain may be defined for various levels of effective strain rate.

In the implementation of this material model, the deviatoric stresses are updated elastically (see material model 1), the yield function is checked, and if it is satisfied the deviatoric stresses are accepted. If it is not, an increment in plastic strain is computed:

$$\Delta \varepsilon_{\text{eff}}^p = \frac{\left( \frac{3}{2} s_{ij}^* s_{ij}^* \right)^{\frac{1}{2}} - \sigma_y}{3G + E_p} \quad (19.10.4)$$

is the shear modulus and $E_p$ is the current plastic hardening modulus. The trial deviatoric stress state $s_{ij}^*$ is scaled back:

$$s_{ij}^{n+1} = \frac{\sigma_y}{\left( \frac{3}{2} s_{ij}^* s_{ij}^* \right)^{\frac{1}{2}}} s_{ij}^* \quad (19.10.5)$$

For shell elements, the above equations apply, but with the addition of an iterative loop to solve for the normal strain increment, such that the stress component normal to the mid surface of the shell element approaches zero.

Three options to account for strain rate effects are possible:

I. Strain rate may be accounted for using the Cowper-Symonds model which scales the yield stress with the factor
\[ \beta = 1 + \left( \frac{\dot{\varepsilon}}{C} \right)^\gamma \] (19.24.2)

where \( \dot{\varepsilon} \) is the strain rate.

II. For complete generality a load curve, defining \( \beta \), which scales the yield stress may be input instead. In this curve the scale factor versus strain rate is defined.

III. If different stress versus strain curves can be provided for various strain rates, the option using the reference to a table definition can be used. See Figure 19.24.1.

A fully viscoplastic formulation is optional which incorporates the different options above within the yield surface. An additional cost is incurred over the simple scaling but the improvement is results can be dramatic.

If a table ID is specified a curve ID is given for each strain rate, see Section 23. Intermediate values are found by interpolating between curves. Effective plastic strain versus yield stress is expected. If the strain rate values fall out of range, extrapolation is not used; rather, either the first or last curve determines the yield stress depending on whether the rate is low or high, respectively.

**Material Model 25: Kinematic Hardening Cap Model**

The implementation of an extended two invariant cap model, suggested by Stojko [1990], is based on the formulations of Simo, et al. [1988, 1990] and Sandler and Rubin [1979]. In this model, the two invariant cap theory is extended to include nonlinear kinematic hardening as suggested by Isenberg, Vaughn, and Sandler [1978]. A brief discussion of the extended cap model and its parameters is given below.

The cap model is formulated in terms of the invariants of the stress tensor. The square root of the second invariant of the deviatoric stress tensor, \( \sqrt{J_{2D}} \) is found from the deviatoric stresses \( \mathbf{s} \) as

\[ \sqrt{J_{2D}} = \sqrt{\frac{1}{2} s_y s_y} \]

and is the objective scalar measure of the distortional or shearing stress. The first invariant of the stress, \( J_1 \), is the trace of the stress tensor.